Chapter 1 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 1: Limits and Continuity.

1.1: Defining a Limit
1.2: Evaluating Limits Analytically
1.3: Squeeze Theorem and Trigonometric Limits
1.4: Continuity
1.5: Formal Definition of a Limit
1.6: Limits with Infinity

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Limits and Continuity

Number of Questions—50

NO CALCULATOR

1. If $\lim_{x\to 3} f(x) = 8$, then which option is true?

(A) $f(3) = 8$.

(B) As x approaches 3, $f(x)$ approaches 8.

(C) $f(8) = 3$.

(D) As x approaches 8, $f(x)$ approaches 3.

(E) f is continuous at $x = 3$.

2. $\lim_{x\to\infty} \frac{8}{x^6}$ is

(A) -8 (B) -1 (C) 0 (D) 1 (E) 8

3. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$ is

(A) 0 (B) $\dfrac{1}{3}$ (C) 1 (D) 3 (E) nonexistent

4. $\lim\limits_{x\to -7}\dfrac{x+7}{49-x^2}$ is

(A) -7 (B) $-\dfrac{1}{14}$ (C) $-\dfrac{1}{49}$ (D) $\dfrac{1}{14}$ (E) nonexistent

5. $\lim\limits_{x\to 0} \dfrac{x}{\tan x}$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

6. $\lim\limits_{x\to \infty} \dfrac{3x^4 - 8x^3 + x^2 - 10}{2x^2 - 5x^3 - 10x^4}$ is

(A) $-\dfrac{3}{10}$ (B) 0 (C) $\dfrac{3}{10}$ (D) $\dfrac{3}{2}$ (E) nonexistent

7. The horizontal asymptote of $f(x) = \dfrac{3x^3 + x^2 - 4}{8 - 5x^3}$ is

(A) $y = -\dfrac{5}{3}$ (B) $y = -\dfrac{3}{5}$ (C) $y = \dfrac{3}{8}$ (D) $y = \dfrac{3}{5}$ (E) $y = \dfrac{5}{3}$

8. $\lim\limits_{x \to \pi/2} \tan 2x$ is

(A) -1 (B) $-\dfrac{1}{2}$ (C) 0 (D) 1 (E) π

9. Let $v(x) = \dfrac{x+3}{x^3+x^2-6x}$. Which option states all the vertical asymptotes of $y = v(x)$?

I. $x = -3$

II. $x = 0$

III. $x = 2$

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

10. Function g is discontinuous at $x = 5$. Selected values of g are shown in the table below.

x	4.99	4.999	5	5.0001	5.001
$g(x)$	2.99	2.999	-4	3.001	3.01

A reasonable estimate for $\lim_{x\to 5} g(x)$ is

(A) -4 (B) -3 (C) 2 (D) 3 (E) 5

11. If $f(x) = \begin{cases} 3x - 2\cos x & x < \pi \\ x^2 & x \geqslant \pi, \end{cases}$ then $\lim_{x\to\pi^-} f(x)$ is

(A) $-\pi^2$ (B) 3π (C) $3\pi - 2$ (D) $3\pi + 2$ (E) π^2

12. $\lim\limits_{x\to 3} \dfrac{\sqrt{x-2}-1}{9-3x}$ is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{6}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{2}$ (E) nonexistent

13. Given that $\lim\limits_{x\to a} f(x)$ exists, which statements must be true?

I. $f(x)$ is continuous at $x = a$.

II. $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$.

III. $f(a)$ is defined.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

14. If $\lim_{k\to 0}\dfrac{e^k-1}{k}=1$, then $\lim_{k\to 0}\dfrac{e^2-e^{2+k}}{k}$ is

(A) $-e^2$ (B) $-e^{-2}$ (C) e^{-2} (D) 1 (E) e^2

15. $g(x)=\dfrac{x^2-5x+6}{x-3}$ has a removable discontinuity at

(A) $x=-3$ (B) $x=-2$ (C) $x=0$ (D) $x=2$ (E) $x=3$

16. $\lim\limits_{x\to\infty} \dfrac{\sin(2x)}{x+2}$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) ∞

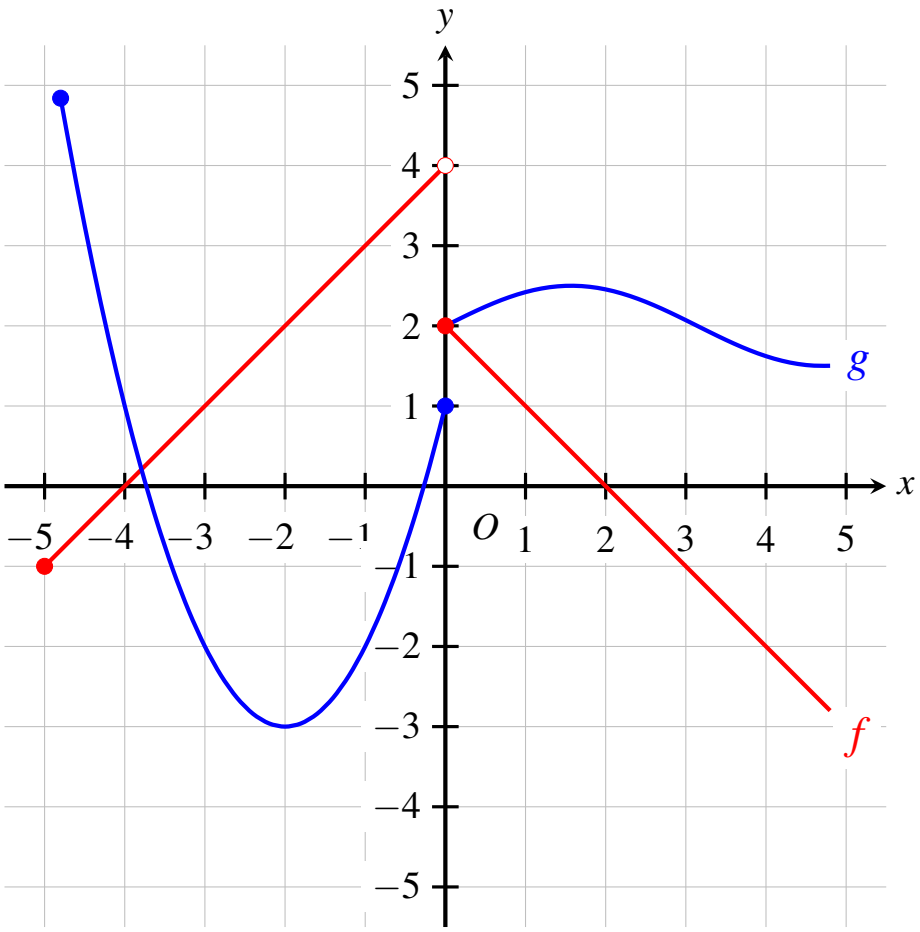
17. Let $f(x) = \begin{cases} 2-kx & x \leqslant 3 \\ kx^2-22 & x > 3. \end{cases}$ For what value of k is f continuous at $x = 3$?

(A) -4 (B) 0 (C) 1 (D) 2 (E) 3

18. $\lim\limits_{x\to\infty} \dfrac{\sqrt{4x^2-1}}{x+3}$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) nonexistent

Questions 19–24 refer to the following graph.



19. $\lim_{x \to 0} f(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

20. $\lim_{x \to 0^+} g(x)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

21. $\lim_{x \to -2} [f(x) - g(x)]$ is

(A) -5 (B) -1 (C) 1 (D) 5 (E) nonexistent

22. $\lim_{x\to -2}[g(x)]^2$ is

(A) -4 (B) -2 (C) 2 (D) 4 (E) 9

23. $\lim_{x\to 0}[f(x)g(x)]$ is

(A) 1 (B) 2 (C) 4 (D) 8 (E) nonexistent

24. $\lim_{x\to 0} f\left(-x^2\right)$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

25. The function $f(x) = \dfrac{\ln(x-2)}{x^2-9}$ is continuous on

(A) $(-\infty,-3)\cup(3,\infty)$

(B) $(-\infty,-2)$

(C) $(2,\infty)$

(D) $(3,\infty)$

(E) $(2,3)\cup(3,\infty)$

26. On $(2,6)$, a function k satisfies

$$1+\frac{8-2x}{x} \leqslant k(x) \leqslant 1+(x-4)\cos 3x\,.$$

Then $\lim\limits_{x\to 4} k(x)$ is

(A) -4 (B) -1 (C) 0 (D) 1 (E) nonexistent

27. $\lim\limits_{x\to -3} \dfrac{5\sin(x+3)}{6+2x}$ is

(A) 0 (B) $\dfrac{5}{6}$ (C) 1 (D) $\dfrac{5}{2}$ (E) nonexistent

28. Function f is continuous and satisfies $f(4) = 8$. If $\lim_{x\to 2} g(x) = 4$, then $\lim_{x\to 2} f(g(x))$ is

(A) -8 (B) -4 (C) 2 (D) 4 (E) 8

29. $\lim_{x\to 2} \dfrac{\frac{1}{2} - \frac{1}{x}}{2-x}$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) nonexistent

30. $\lim\limits_{x\to 0} x\sin\left(\dfrac{1}{x^2}\right)$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E) nonexistent

31. Let $f(x)=\begin{cases} x^2+1 & x<p \\ 2x & x\geqslant p \end{cases}$. If $f(x)$ is continuous at $x=p$, then p is

(A) -1 (B) 0 (C) 1 (D) 2 (E) 4

32. If $\lim_{x\to 3}(2x+4)=10$, then $|(2x+4)-10|<\varepsilon$ and $0<|x-3|<\delta$, where $\delta=$

(A) $\frac{\varepsilon}{4}$ (B) $\frac{\varepsilon}{2}$ (C) ε (D) 2ε (E) 4ε

33. $\lim_{t\to\infty}\sin(5t-5)$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

34. If $\lim\limits_{x\to 2} f(x) = -3$, then $\lim\limits_{x\to 2} \left([f(x)]^2 - 2x\right)$ is

(A) -7 (B) -3 (C) 4 (D) 5 (E) 9

35. Function f is continuous. Selected values of $f(x)$ are shown in the table below.

x	-1	2	3	6	11
$f(x)$	2	1	1	1	-1

Following the Intermediate Value Theorem, which value of $f(x)$ must exist on $[-1, 11]$?

(A) 0 (B) 3 (C) 5 (D) 6 (E) 11

36. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{4x^6-4x^2+1}}{3x^3+2}$ is

(A) $-\infty$ (B) $-\dfrac{2}{3}$ (C) $\dfrac{2}{3}$ (D) 1 (E) ∞

37. $\lim\limits_{x\to\infty} x\sin\left(\dfrac{1}{x}\right)$ is

(A) -1 (B) 0 (C) 1 (D) π (E) nonexistent

38. At $x = 4$, which choice about $g(x) = \dfrac{12 + x - x^2}{x - 4}$ is true?

(A) $g(x)$ has a vertical asymptote at $x = 4$.

(B) $g(x)$ has a jump discontinuity at $x = 4$.

(C) $g(x)$ has a removable discontinuity at $x = 4$.

(D) $\lim\limits_{x \to 4} g(x)$ does not exist.

(E) $g(x)$ is continuous at $x = 4$.

39. $\lim\limits_{x \to 0^+} \ln(\sin x)$ is

(A) $-\infty$ (B) 0 (C) 1 (D) e (E) ∞

40. $\lim\limits_{x\to 0} \dfrac{x^2}{\sin^2 2x}$ is

(A) 0 (B) $\dfrac{1}{4}$ (C) 1 (D) 4 (E) nonexistent

41. Consider the rational function $f(x) = \dfrac{x^{2p}+4}{7x^q-6}$, where p and q are positive real numbers. Which choice lists *all* the cases of p and q for which $\lim\limits_{x\to\infty} f(x)$ is finite?

(A) $p \leqslant \dfrac{q}{2}$ (B) $p < \dfrac{q}{2}$ (C) $p = \dfrac{q}{2}$ (D) $p > \dfrac{q}{2}$ (E) $p \geqslant \dfrac{q}{2}$

42. $\lim\limits_{x\to 0} \dfrac{x - x\cos x}{x^2}$ is

(A) $-\pi$ (B) 0 (C) 1 (D) π (E) nonexistent

43. $\lim\limits_{x\to 0^+} \arctan\left(\dfrac{1}{x}\right)$ is

(A) $-\infty$ (B) $-\dfrac{\pi}{2}$ (C) 0 (D) $\dfrac{\pi}{2}$ (E) ∞

44. $\lim\limits_{t\to\infty} \dfrac{1}{\ln t} = 0$ means that for every $\varepsilon > 0$, there exists a number N such that $\left|\dfrac{1}{\ln t} - 0\right| < \varepsilon$ if $t > N$, where $N =$

(A) $\dfrac{1}{\varepsilon}$ (B) $e^{-\varepsilon}$ (C) $e^{-1/\varepsilon}$ (D) $e^{1/\varepsilon}$ (E) e^{ε}

45. $\lim\limits_{x\to\infty} \dfrac{5e^x - x}{8e^x + 9}$ is

(A) $-\dfrac{5}{8}$ (B) 0 (C) $\dfrac{5}{8}$ (D) 1 (E) ∞

46. $\lim\limits_{x\to\pi/4} \dfrac{\cos 2x}{\cos x - \sin x}$ is

(A) 0 (B) $\dfrac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) π (E) nonexistent

47. Functions g and h are continuous and satisfy $g(1) = h(1) = 3$. Function f satisfies $g(x) \leqslant f(x) \leqslant h(x)$ for $0 \leqslant x \leqslant 2$. Which statements must be true?

I. $\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1} h(x) = 3$.

II. $\lim\limits_{x\to 1} f(x) = 3$.

III. $f(x)$ is continuous at $x = 1$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

48. If $f(1)=2$, $\lim\limits_{x\to 1^-} f(x)=4$, and $\lim\limits_{x\to 1^+} f(x)=-1$, then $\lim\limits_{x\to 1} f(\cos(x-1))$ is

(A) -1 (B) 0 (C) 2 (D) 4 (E) nonexistent

49. Which option is a correct proof of $\lim\limits_{x\to 1^+} \dfrac{1}{\sqrt{x-1}} = \infty$?

(A) For every positive M, there exists a number δ such that $\dfrac{1}{\sqrt{x-1}} > M$ for $1-\delta < x < 1$, where $\delta = \dfrac{1}{\sqrt{M}}$.

(B) For every positive M, there exists a number δ such that $\dfrac{1}{\sqrt{x-1}} > M$ for $1-\delta < x < 1$, where $\delta = \dfrac{1}{M^2}$.

(C) For every positive M, there exists a number δ such that $\dfrac{1}{\sqrt{x-1}} > M$ for $0 < |x-1| < \delta$, where $\delta = \dfrac{1}{M^2}$.

(D) For every positive M, there exists a number δ such that $\dfrac{1}{\sqrt{x-1}} > M$ for $1 < x < 1+\delta$, where $\delta = \dfrac{1}{\sqrt{M}}$.

(E) For every positive M, there exists a number δ such that $\dfrac{1}{\sqrt{x-1}} > M$ for $1 < x < 1+\delta$, where $\delta = \dfrac{1}{M^2}$.

50. For what values of k does the graph of $p(x) = \dfrac{x^2 - 2kx + 16}{x - k}$ have no vertical asymptotes?

I. $k = -4$

II. $k = 0$

III. $k = 4$

IV. $k = 16$

(A) I and II only

(B) I and III only

(C) II and IV only

(D) I, II, and III only

(E) I, II, III, and IV

This marks the end of the test. The following page contains the answers to all the questions.

1. B
2. C
3. D
4. D
5. C
6. A
7. B
8. C
9. D
10. D
11. D
12. B
13. B
14. A
15. E
16. B
17. D
18. C
19. E
20. C
21. D
22. E
23. C
24. D
25. E
26. D
27. D
28. E
29. B
30. A
31. C
32. B
33. E
34. D
35. A
36. B
37. C
38. C
39. A
40. B
41. A
42. B
43. D
44. D
45. C
46. C
47. E
48. D
49. E
50. B