Chapter 10 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 10: Infinite Sequences and Series.

10.1: Sequences
10.2: Infinite Series and Divergence Test
10.3: Integral Test
10.4: Comparison Tests
10.5: Alternating Series
10.6: Absolute Convergence and the Ratio and Root Tests
10.7: Power Series
10.8: Taylor and Maclaurin Series

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Infinite Sequences and Series

Number of Questions—50

NO CALCULATOR

1. What is the difference between a sequence and a series?

(A) A sequence is the sum of a list of numbers, whereas a series is a list of numbers.

(B) A sequence is a list of numbers arranged in ascending order, whereas a series is the sum of a list of numbers arranged in ascending order.

(C) A sequence is a list of numbers, whereas a series is the sum of a list of numbers.

(D) A sequence always diverges, whereas a series may converge or diverge.

(E) A sequence always converges, whereas a series may converge or diverge.

2. Which sequence converges?

(A) $\{-1,1,-1,1,-1,\dots\}$

(B) $\{-2,-1,0,1,2,\dots\}$

(C) $\{1,2,4,8,16,\dots\}$

(D) $\left\{1,\frac{3}{4},\frac{9}{16},\frac{27}{64},\frac{81}{256},\dots\right\}$

(E) $\left\{\frac{1}{2},-\frac{3}{2},\frac{5}{2},-\frac{7}{2},\frac{9}{2},\dots\right\}$

3. $\frac{\pi}{4}-\frac{\pi^2}{16}+\frac{\pi^3}{64}-\frac{\pi^4}{256}+\cdots+(-1)^{n+1}\left(\frac{\pi}{4}\right)^n+\cdots$ is

(A) $\frac{-4}{4+\pi}$ (B) $\frac{\pi}{4-\pi}$ (C) $\frac{\pi}{4+\pi}$ (D) $\frac{4}{4-\pi}$ (E) divergent

4. Which sequence is monotonic?

I. $a_n = \dfrac{2}{\sqrt{n+4}}$.

II. $b_n = \dfrac{(-1)^n}{n^7+1}$

III. $c_n = \dfrac{2n^2+8}{n^2+4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III

5. The Maclaurin series of $\cos x$ is

(A) $1+\frac{x^2}{2}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$

(B) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$

(C) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots+\frac{x^{2n+1}}{(2n+1)!}+\cdots$

(D) $1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$

(E) $1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$

6. Let $a_n = \dfrac{1}{\sqrt[3]{n^2}}$. Which option is true?

(A) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(B) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges because it is a p-series with $p \leqslant 1$.

(C) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

(D) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges because it is a p-series with $p \leqslant 1$.

(E) It cannot be determined from the given information.

7. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}}$ (B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ (C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ (D) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ (E) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2\sqrt{n}}$

8. Suppose that $a_n \geqslant b_n > 0$ and $\{b_n\}$ diverges. Which of the following must be true?

I. $\{a_n\}$ diverges.

II. $\lim_{n\to\infty} a_n = \infty$.

III. $\lim_{n\to\infty} b_n = \infty$.

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

9. Which series is alternating?

(A) $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n}$

(B) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\cos(\pi n)$

(C) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{2n+3}}{n}$

(D) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(\pi n)}{\sqrt{n}}$

(E) All of the above

10. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Which of the following must be true?

I. $\lim_{n\to\infty} a_n = 0$

II. The sequence $\{a_n\}$ converges.

III. $\lim_{N\to\infty} \sum_{n=1}^{N} a_n = \pm\infty$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

11. The Maclaurin series $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots$ converges to which function?

(A) $\sin x$ (B) $\cos x$ (C) $x^2 e^x$ (D) $x^2 \sin x$ (E) $x \sin x$

12. Which series converges?

(A) $\sum_{n=1}^{\infty}(-1)^n$ (B) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ (C) $\sum_{n=1}^{\infty}(-1)^n n$ (D) $\sum_{n=1}^{\infty}(-1)^{2n}$ (E) $\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n}$

13. Suppose that $\sum_{n=1}^{\infty} b_n$ converges and $a_n \leqslant b_n$ for $n \geqslant 1$. Which choice must be true?

(A) $\sum_{n=1}^{\infty} a_n$ converges.

(B) $\sum_{n=1}^{\infty} a_n$ diverges.

(C) $\lim_{n\to\infty} a_n = 0$.

(D) $\lim_{n\to\infty} b_n \neq 0$.

(E) None of the above

14. Function g has derivatives of all orders. The table below shows selected values of the derivatives of $g(x)$ at $x = 3$, with $g(3) = 1$. What is the fourth-degree Taylor polynomial for $g(x)$ centered at $a = 3$?

n	1	2	3	4
$g^{(n)}(3)$	-2	2	$3/2$	$-4/3$

(A) $1-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(B) $1-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(C) $-2(x-3)+2(x-3)^2+\frac{3}{2}(x-3)^3-\frac{4}{3}(x-3)^4$

(D) $-2(x-3)+(x-3)^2+\frac{1}{4}(x-3)^3-\frac{1}{18}(x-3)^4$

(E) $1-2(x+3)+(x+3)^2+\frac{1}{4}(x+3)^3-\frac{1}{18}(x+3)^4$

15. For what values of k does $\sum_{n=1}^{\infty} \frac{1}{n^{2k-3}}$ diverge?

(A) $k \leqslant 2$

(B) $k < \frac{3}{2}$

(C) $k > \frac{3}{2}$

(D) $k > 2$

(E) $k \geqslant 2$

16. The Taylor polynomial $T_N(x)$ is centered at a and is used to approximate $f(x)$. Which statement is true about Taylor's Remainder Theorem?

(A) Taylor's Remainder Theorem uses the maximum value of $\left|f^{(N)}(x)\right|$ on some closed interval containing a.

(B) Taylor's Remainder Theorem is only applicable to non-alternating series.

(C) Taylor's Remainder Theorem enables the calculation of the exact error in $T_N(x)$.

(D) Taylor's Remainder Theorem bounds the remainder of $T_N(x)$ by the magnitude of the first omitted term of the Taylor approximation, that is, the $(N+1)$st term.

(E) Taylor's Remainder Theorem gives a value greater than or equal to the true error in $T_N(x)$.

17. Which choice correctly describes the convergence or divergence of $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}}$?

(A) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(B) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(C) The series converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) The series diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(E) It cannot be determined from the given information.

18. Let f be a function with derivatives of all orders. The fourth-degree Taylor polynomial of $f(x)$ centered at $a = 2$ is $2+(x-2)-\frac{2}{5}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{1}{7}(x-2)^4$. The value of $f'''(2)$ is

(A) -9 (B) $-\frac{9}{2}$ (C) $-\frac{3}{2}$ (D) $-\frac{1}{4}$ (E) 0

19. The coefficient of x^3 in the Maclaurin series of $e^{-x/3}$ is

(A) $-\dfrac{1}{27}$ (B) $-\dfrac{1}{162}$ (C) $\dfrac{1}{162}$ (D) $\dfrac{1}{27}$ (E) $\dfrac{1}{6}$

20. Which series converges conditionally?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+1}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+2}}$

21. Which choice about $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+3}}$ is correct?

(A) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ converges.

(B) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ diverges.

(C) The series converges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx > 0$.

(D) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ converges.

(E) The series diverges because $\int_1^{\infty} \frac{1}{\sqrt{2x+3}}\,dx$ diverges.

22. The binomial series expansion of $\frac{1}{2}\sqrt{4-x}$ is

(A) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(B) $1-\frac{1}{2}\left(-\frac{x}{4}\right)-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2-\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(C) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

(D) $1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\cdots$

(E) $1+\frac{1}{2}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\cdots$

23. For which series is the Root Test inconclusive?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{8^n+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+3}{n+2}\right)^n$

(C) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+7}{3n-1}\right)^n$

(D) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6n^2+n-2}{4n^2+9}\right)^n$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{5^{3n+1}}$

24. The radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{4n+2}}{n^6}$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

25. Suppose that $\sum_{n=1}^{\infty} a_n$ converges and c is a positive integer. All the following series must converge *except*

(A) $c + \sum_{n=1}^{\infty} a_n$

(B) $c \sum_{n=1}^{\infty} a_n$

(C) $\sum_{n=1+c}^{\infty} a_n$

(D) $\sum_{n=1}^{\infty} (a_n)^c$

(E) $\sum_{n=1}^{\infty} (a_n + c\,a_n)$

26. The fourth-degree Taylor polynomial of e^x centered at $a = 2$ is

(A) $1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^4$

(B) $1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^4$

(C) $e^2\left[1+(x+2)+\frac{1}{2}(x+2)^2+\frac{1}{3!}(x+2)^3+\frac{1}{4!}(x+2)^4\right]$

(D) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3!}(x-2)^3+\frac{1}{4!}(x-2)^4\right]$

(E) $e^2\left[1+(x-2)+\frac{1}{2}(x-2)^2+\frac{1}{3}(x-2)^3+\frac{1}{4}(x-2)^4\right]$

27. Which function does *not* have a Taylor series at the given center?

(A) $|x|$ centered at $a = 0$

(B) e^x centered at $a = e$

(C) $\tan x$ centered at $a = \frac{\pi}{3}$

(D) $\sqrt{x}$ centered at $a = 1$

(E) $\ln x$ centered at $a = 1$

28. The radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n n^2}{n!}$ is

(A) 0 (B) 1 (C) 2 (D) e (E) ∞

29. Suppose that $a_n \geqslant b_n \geqslant 0$ and $\sum_{n=1}^{\infty} a_n$ diverges. Which of the following must be true?

I. $\sum_{n=1}^{\infty} b_n$ diverges.

II. $\lim_{N\to\infty} \sum_{i=1}^{N} a_i = \infty$.

III. $\lim_{N\to\infty} b_N = 0$.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

30. The third-degree term in the binomial series for $\sqrt[3]{54x+27}$ is

(A) $\dfrac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{2}\left(\dfrac{x}{2}\right)^3$

(B) $\dfrac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{2}(2x)^3$

(C) $\dfrac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(2x)^3$

(D) $\dfrac{\frac{1}{3}\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)}{3!}\left(\dfrac{x}{2}\right)^3$

(E) $\dfrac{\frac{1}{3}\left(\frac{4}{3}\right)\left(\frac{7}{3}\right)}{3!}(2x)^3$

31. For $-1 < x < 1$, which series is equivalent to $\dfrac{1}{1-x^2}$?

(A) $x^2 - x^4 + x^6 - x^8 + \cdots$

(B) $x^2 + x^4 + x^6 + x^8 + \cdots$

(C) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(D) $1 - x^2 + x^4 - x^6 + x^8 + \cdots$

(E) $-1 - x^2 - x^4 - x^6 - x^8 - \cdots$

32. Which series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^3+9}}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5}}{\sqrt{n^6+9}}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{3n^4+4}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+2}}$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{2n+1}}{\sqrt{5n^4+3}}$

33. Let $a_n = \dfrac{n^2\sqrt{n^2+4}}{n^3}$. Which choice is correct?

(A) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test for $\sum\limits_{n=1}^{\infty} a_n$ is inconclusive.

(B) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| > 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(C) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| < 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

(D) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| < 1$, $\sum\limits_{n=1}^{\infty} a_n$ converges by the Ratio Test.

(E) Because $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1$, $\sum\limits_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

34. Function f has derivatives of all orders. At $x = 0$, $f(x)$ is decreasing and concave up. Which choice could be the third-degree Maclaurin polynomial for f ?

(A) $2+x-\frac{1}{3}x^2+4x^3$

(B) $2+x+\frac{1}{3}x^2-4x^3$

(C) $-2-x-\frac{1}{3}x^2-4x^3$

(D) $-2-x+\frac{1}{3}x^2-4x^3$

(E) $2+x+\frac{1}{3}x^2+4x^3$

35. Let $a_n = \dfrac{1}{\sqrt{n+2}}$ for $n \geqslant 0$. Let f be a positive, continuous, decreasing function such that $f(n) = a_n$. Which choice must be true?

(A) $\displaystyle\sum_{n=2}^{\infty} a_n \leqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(B) $\displaystyle\sum_{n=2}^{\infty} a_n \geqslant \int_1^{\infty} f(x)\,\mathrm{d}x$

(C) $\displaystyle\sum_{n=2}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(D) $\displaystyle\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x)\,\mathrm{d}x$

(E) It cannot be determined from the given information.

36. $\dfrac{d}{dx}\displaystyle\sum_{n=1}^{\infty}\frac{x^{3n+1}}{n!} =$

(A) $\dfrac{x^5}{5}+\dfrac{x^8}{8(2!)}+\dfrac{x^{11}}{11(3!)}+\cdots+\dfrac{x^{3n+2}}{(3n+2)n!}+\cdots$

(B) $4x+\dfrac{7}{2!}x^4+\dfrac{10}{3!}x^7+\cdots+(3n+1)\dfrac{3^{3n-2}}{n!}+\cdots$

(C) $\dfrac{x^7}{7}+\dfrac{x^{10}}{10(2!)}+\dfrac{x^{13}}{13(3!)}+\cdots+\dfrac{x^{3n+4}}{(3n+4)n!}+\cdots$

(D) $4x^3+\dfrac{1}{2!}x^6+\dfrac{1}{3!}x^9+\cdots+\dfrac{x^{3n}}{n!}+\cdots$

(E) $4x^3+\dfrac{7}{2!}x^6+\dfrac{10}{3!}x^9+\cdots+(3n+1)\dfrac{x^{3n}}{n!}+\cdots$

37. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3n^2+3n}\right)$ is

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$ (E) divergent

38. Let $S = \sum_{n=1}^{\infty} \frac{\sin 8n}{5n^2}$. Which of the following must be true?

I. The series converges absolutely.

II. The series converges conditionally.

III. The series converges.

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

39. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!}$. What is the coefficient of the x^6 term in $\int_0^x f(t)\,\mathrm{d}t$?

(A) $-\frac{7}{6}$ (B) $-\frac{1}{12}$ (C) $-\frac{1}{48}$ (D) $\frac{1}{12}$ (E) $\frac{7}{6}$

40. What is the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$?

(A) The series diverges for all x.

(B) $-1 < x < 1$

(C) $-1 \leqslant x \leqslant 1$

(D) $0 < x < 1$

(E) $0 \leqslant x \leqslant 1$

41. Let $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+4}}{n^2}$. The coefficient of the x^6 term in $f'(x)$ is

(A) $-\dfrac{7}{9}$ (B) $-\dfrac{1}{4}$ (C) $-\dfrac{1}{24}$ (D) $\dfrac{1}{6}$ (E) $\dfrac{7}{9}$

42. Which series test is appropriate to test $\sum_{n=1}^{\infty} \frac{1}{n\,e^n}$ for convergence or divergence, where $a_n = \frac{1}{n\,e^n}$?

(A) The Integral Test, by comparing $\sum_{n=1}^{\infty} a_n$ to $\int_1^{\infty} \frac{1}{x\,e^x}\,\mathrm{d}x$

(B) The Limit Comparison Test, by considering $\lim_{n\to\infty} \frac{a_n}{b_n}$ with $b_n = \frac{1}{n}$

(C) The Limit Comparison Test, by considering $\lim_{n\to\infty} \frac{a_n}{b_n}$ with $b_n = \frac{1}{e^n}$

(D) The Direct Comparison Test, by considering $0 \leqslant a_n \leqslant b_n$ with $b_n = \frac{1}{n}$

(E) The Direct Comparison Test, by considering $0 \leqslant a_n \leqslant b_n$ with $b_n = \frac{1}{e^n}$

43. Function g has derivatives of all orders. The fourth-degree Maclaurin polynomial of $g(x)$ is $m(x)$. It is known that $|g^{(5)}(x)| \leqslant 0.7$ for $0 \leqslant x \leqslant 0.4$. Which choice must be true?

(A) $|g(0.4) - m(0.4)| \leqslant 0.7$

(B) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{4!}(0.4)^4$

(C) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{4!}(0.4)^4$

(D) $|g(0.4) - m(0.4)| \geqslant \dfrac{0.7}{5!}(0.4)^5$

(E) $|g(0.4) - m(0.4)| \leqslant \dfrac{0.7}{5!}(0.4)^5$

44. The power series $\sum_{n=0}^{\infty} c_n(x-5)^n$ converges at $x=7$ and diverges at $x=8$. Which of the following must be true?

I. The series converges at $x=4$.

II. The series converges at $x=3$.

III. The series diverges at $x=2$.

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

45. The power series $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ has a radius of convergence of R. The power series of which of the following must also have a radius of convergence of R ?

I. $f'(x)$

II. $\int f(x)\,\mathrm{d}x$

III. $f(2x)$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

46. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+8}}$. Using the Alternating Series Error Bound, what is the least number of terms that must be added to guarantee a partial sum that is within $\frac{1}{30}$ of S ?

(A) 22 (B) 30 (C) 891 (D) 900 (E) 908

47. A function f has derivatives of all orders, and a Taylor series for f centered at $a = 6$ is

$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}(x-6)^{n+2}}{n^2}$. Then $f^{(4)}(6) =$

(A) -24 (B) -6 (C) $-\frac{1}{6}$ (D) $-\frac{1}{24}$ (E) 6

48. The third nonzero term of the Maclaurin series of $48x^2 \cos\left(\frac{x}{2}\right)$ is

(A) $\frac{x^4}{16}$ (B) $\frac{x^4}{8}$ (C) $\frac{x^6}{32}$ (D) $\frac{x^6}{16}$ (E) $\frac{x^6}{8}$

49. The partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^3}$ is used to estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Based on the Integral Test, what is the smallest value of k such that the error in S_k is no greater than $\frac{1}{200}$?

(A) 2 (B) 10 (C) 50 (D) 100 (E) 200

50. Let $f(x) = \dfrac{x}{1+x^6}$. Let $g(x) = \displaystyle\int f(x)\,\mathrm{d}x$ and $g(0) = 1$. What is the Maclaurin series expansion of g ?

(A) $\dfrac{x^2}{2} - \dfrac{x^8}{8} + \dfrac{x^{14}}{14} - \dfrac{x^{20}}{20} + \cdots + \dfrac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(B) $x - x^7 + x^{13} - x^{19} + \cdots + (-1)^n x^{6n+1} + \cdots$

(C) $1 + x - x^7 + x^{13} + \cdots + (-1)^n x^{6n+1} + \cdots$

(D) $1 + \dfrac{x^2}{2} - \dfrac{x^8}{8} + \dfrac{x^{14}}{14} + \cdots + \dfrac{(-1)^n x^{6n+2}}{6n+2} + \cdots$

(E) $1 - 7x^6 + 13x^{12} - 19x^{18} + \cdots + (-1)^n (6n+1) x^{6n} + \cdots$

This marks the end of the test. The following page contains the answers to all the questions.

1. C
2. D
3. C
4. D
5. E
6. D
7. C
8. A
9. D
10. C
11. E
12. B
13. E
14. A
15. A
16. E
17. D
18. A
19. B
20. C
21. E
22. A
23. B
24. C
25. D
26. D
27. A
28. E
29. B
30. B
31. C
32. C
33. A
34. D
35. A
36. E
37. B
38. C
39. D
40. C
41. E
42. E
43. E
44. A
45. D
46. C
47. B
48. E
49. B
50. D